Roll No. ....

Total No. of Questions : 9] [Total No. of Pages : 03

MCA (Sem. - 2nd)

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

SUBJECT CODE : MCA-201 (2012 Batch)

Paper ID : [B0133]

Time : 03 Hours **Maximum Marks : 100**

Instruction to Candidates:

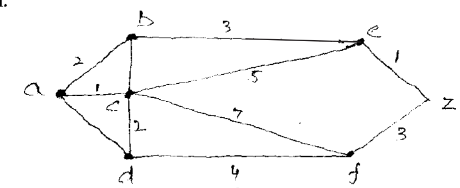
1) Attempt one question each from sections A, B, C and D.
2) Section E is compulsory.

Section - A

(1 × 20 = 20)

Q1) a) Prove that a simple graph with k-components and n-vertices can have at the most $\frac{(n-k)(n-k+1)}{2}$ edges.

b) Find the shortest path between a and z in given graph using Dijkstra's algorithm.



OR

Q2) a) State and prove Euler's theorem for planar graph.

b) Give an example of a graph which has an Euler's circuit but not a Hamiltonian circuit.

Section - B

(1 × 20 = 20)

Q3) a) Consider A = {1, 2, 3, 4, 5, 6, 7, 8, 9} and let B_1 = {5, 6, 7}, B_2 = {2, 4, 5, 9}, B_3 = {3, 4, 5, 6, 8, 9}. Find the minsets generated by B_1, B_2, B_3.

b) Consider the set A = {7, 8, 9}. Then find all the partitions of the set A.

OR

Q4) a) If R and S are two equivalence relations on a set A then show that $R \cap S$ is also an equivalence relation on set A and $R \cup S$ may or may not be an equivalence relation on A.

b) Let A, B and C be three sets and R be a relation from A to B and S be a relation from B to C. Then show that $(S_o R)^{-1} = R^{-1}{}_o S^{-1}$.

Section - C

(1 × 20 = 20)

Q5) Determine whether, the following are equivalent, using Biconditional statement.

i) $p \leftrightarrow q \cong (p \wedge q) \vee (\sim p \wedge q \sim)$

ii) $(p \to q) \to t \cong (p \wedge \sim q) \to t$

OR

Q6) a) Prove by induction that the sum of the cubes of three consecutive integers is divisible by 9.

b) Prove that the negation of biconditional statement $\sim (p \leftrightarrow q)$ is equivalent to $p \leftrightarrow \sim q$ or $\sim p \leftrightarrow q$.

Section - D

(1 × 20 = 20)

Q7) a) If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ then find A^{-1} and also show that $A^3 = A^{-1}$.

b) Find the rank of the matrix $\begin{pmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{pmatrix}$.

OR

Q8) a) Solve the given simultaneous system of equations $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ using Gauss - Jordan method.

b) For what values of λ and μ do the given system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + z + \lambda z = \mu$, have

i) unique solution

ii) no solution

iii) infinite solutions

Section - E

(10 × 2 = 20)

Q9) a) A graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in G.

b) Define complete Bipartite graph.

c) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

d) Show that every subset of a countable set is countable.

e) Prove that a relation R on a set A is symmetric if and only if $R = R^{-1}$.

f) Prove that the statement $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

g) Express $(P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ in terms of $(\sim, \wedge)$ only.

h) Define Existential and universal Quantifier.

i) Find x, y, z and w if $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$.

j) Show that $\sum \sim, \to 3$ is functionally complete.

⌘⌘⌘