

Roll No _____

Time: 3 hrs.**Max Marks: 100**

Note: Attempt **five** questions in all, including **Q.No. 9 (Section-E)** which is **compulsory** and selecting **one** question each from sections A, B, C and D. All questions carry equal marks.

Section – A

- Q1. (a) What is a bipartite graph and a complete bipartite graph? Draw $K_{3,3}$.
 (b) Show that
- No simple graph can have all the vertices with distinct degrees.
 - If a graph has Euler's path, then it has either no vertex of odd degree or two vertices of odd degree. (10, 10)
- Q2. (a) Discuss the Dijkstra's algorithm to find the shortest path from one specified vertex u to another specified vertex v , with the help of an example.
 (b) State and prove the five-color problem. (10, 10)

Section – B

- Q3. (a) In a town of 10000 families, it was found that 40% families buy newspaper A, 20% buy newspaper B and 10% buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the newspapers, find the number of families which buy (i) A only (ii) B only (iii) none of A, B and C.
 (b) Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n . (10, 10)
- Q4. (a) Partition the set $A = \{1, 2, 3, \dots, 10\}$ using the minsets generated by $B_1 = \{1, 7, 8\}$, $B_2 = \{1, 6, 9, 10\}$, $B_3 = \{1, 9, 10\}$. Also represent the minsets thus generated, through a Venn diagram.
 (b) If A and B are two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.
 (c) If $A = \{1, 2, 3, 4\}$. Consider the following relation in A
 $R = \{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$. Draw its directed graph. (9, 6, 5)

Section – C

- Q5. (a) Verify $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology.
 (b) Construct the truth table of $\sim(p \wedge q) \rightarrow \sim p \vee \sim q$. Is it a contradiction?
 (c) Let p be "He is rich" and q be "He is honest". Write the following statements in symbolic forms using p and q :
- To be poor is to be honest
 - It is necessary to be poor in order to be honest
 - He is poor only if he is dishonest (7, 7, 6)
- Q6. (a) Prove by the method of induction that every even power of every odd number greater than 1 when divided by 8, leaves 1 as a remainder.
 (b) Write the following statements in symbolic form and give their negations:

- (i) If you work hard, you will get the first division.
- (ii) If it rains, he will not go to Kathua.
- (iii) If Mahatma Gandhi was a saint then Sardar Patel was an iron man.

(14, 6)

Section – D

Q7. (a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

- (b) Discuss matrix addition, scalar multiplication and multiplication of matrices by taking some suitable example. (10, 10)

Q8. (a) Solve the following system of equations using Matrix Inversion method:

$$2x - 2y + 5z = 13$$

$$2x + 3y + 4z = 20$$

$$3x - y + 3z = 10$$

- (b) Discuss the steps involved in Gauss-Elimination and Gauss-Jordan method of solving simultaneous equations. (8, 12)

Section – E (Compulsory)

Q9. Attempt and explain the following briefly:

- (i) Differentiate between simple graph and a multi graph.
- (ii) What is chromatic number?
- (iii) What do you mean by Cartesian product?
- (iv) State the DeMorgan's law.
- (v) Differentiate between countable and uncountable sets.
- (vi) Give different types of quantifiers.
- (vii) Explain 'equivalence' and 'implication'.
- (viii) What do you mean by transpose of a matrix?
- (ix) Explain the rank of a matrix.
- (x) What is a proposition?

(10 X 2 = 20)