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Total No. of Questions : 09

MCA (2012 & onward) (Sem.-2)

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

Subject Code : MCA-201

Paper ID : [B0133]

Time : 3 Hrs.

Max. Marks : 100

INSTRUCTION TO CANDIDATES :

- 1. SECTIONS-A, B, C & D contains TWO questions each carrying TWENTY marks each and students has to attempt any ONE question from each SECTION.**
- 2. SECTION-E is COMPULSORY consisting of TEN questions carrying TWENTY marks in all.**
- 3. Use of non-programmable scientific calculator is allowed.**

SECTION-A

1. (a) Explain Dijkstra's algorithm with the help of an example.
(b) Prove that the sum of the degrees of the vertices of any finite graph is even.
(c) Show that every simple graph has two vertices of the same degree. (10,5,5)
2. (a) Differentiate, with example, a simple graph and a multigraph. Show that the maximum number of edges in a simple graph with n vertices $n(n-1)/2$.
(b) Explain eulerian and hamiltonian graph with example. Draw a graph that has a Hamiltonian circuit. (10,10)

SECTION-B

3. (a) A survey of 85 students asked them about the subjects they liked to study. Thirty five students liked math, 37 liked history, and 26 liked physics. Twenty liked math and history, 14 liked math and physics and 3 liked history and physics. Two students liked all three subjects.
- (i) How many of these students like math and physics ?
- (ii) How many of these students didn't like any of the three subjects ?
- (iii) How many of these students liked math and history but not physics ?
- (b) If $H = \{1, 2, 3, 4\}$. Consider the following relation in H .
- $R = \{(1,1), (2,2), (2,3), (3,2), (4,2), (4,4)\}$. Draw its directed graph. **(15,5)**
4. (a) State and prove De Morgan's laws for the two sets A and B .

- (b) Let $A = \{0,2,3\}$, $B = \{2,3\}$, $C=\{1,4\}$ and let the universal set be $U = \{0,1,2,3,4\}$. List the elements of (i) $A \times (B \cup C)$ (ii) B^3 (iii) $A \times A^c$.

SECTION-C

5. (a) Consider the Fibonacci sequence, defined by the relations $x_1 = 1$, $x_2 = 1$, and $x_n = x_{n+1} + x_{n-2}$ for $n \geq 3$. Use Principle of Mathematical Induction in order to show that for $n \geq 1$,

$$x_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

- (b) Translate the following into symbolic form :

- (i) Everybody likes him.
- (ii) Somebody cried out for help and called the police.
- (iii) Nobody can ignore her.

6. (a) Use symbols to write the logical form of the given argument and then use a truth table to test the argument for validity.

If Tom is not on team A, then Hua is on team B.

If Hua is not on team B, then Tom is on team A.

Therefore, Tom is not on team A or Hua is not on team B.

- (b) Show that $(r \vee p) \wedge ((\neg r \vee (p \wedge q)) \wedge (r \vee q)) = p \wedge q$

SECTION-D

7. (a) Find the rank of the following matrix.

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 4 & 0 & -2 & 1 \\ 3 & -1 & 0 & 4 \end{bmatrix}$$

(b) Solve the following system of equations

$$3x + y - 2z = 2$$

$$x - 2y + z = 3$$

$$2x - y - 3z = 3$$

8. (a) Let $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -3 \\ 2 & 1 \end{pmatrix}$. Verify that $(AB)^{-1} = B^{-1} A^{-1}$.

(b) Solve the following system using Gauss-Jordan elimination.

$$3x_1 + x_2 + 7x_3 + 2x_4 = 13$$

$$2x_1 - 4x_2 + 14x_3 - x_4 = -10$$

$$5x_1 + 11x_2 - 7x_3 + 8x_4 = 59$$

$$2x_1 + 5x_2 - 4x_3 - 3x_4 = 39$$

SECTION-E

9. **Write briefly :**

a) Write the negation of the proposition: If an integer is divisible by 2, then it is even.

b) Given that

$A = \{2, 4\}$ and $B = \{x : x \text{ is a solution of } x^2 + 6x + 8 = 0\}$. Are A and B disjoint sets?

c) Explain rectangular and square matrix with example.

d) Let t be a tautology. Show that $p \vee t = t$.

e) Define cycle in a graph.

f) If $A = \{2, 4, 6, 8, 10\}$, $B = \{8, 10, 12, 14\}$, $C = \{14, 16, 18, 20\}$.

Find (i) $A \cap (B \cup C)$ (ii) $(A \cap B) \cup C$

g) Explain directed and undirected graph with example.

h) Show that $AB = BA$ if and only if $(A + B)^2 = A^2 + 2AB + B^2$.

i) Define Equivalence relation with example.

j) Explain bipartite graph.