

Roll No.

--	--	--	--	--	--	--	--	--	--	--	--

Total No. of Pages: 03

Total No. of Questions: 09

**B.Tech (AI&ML / AI and Data Science / CSE / IOT / Data Science / IT / Robotics
& AI / Internet of Things and Cyber Security including Block Chain
Technology / Computer Engg.) (Sem- 1)**

MATHEMATICS-I

Subject Code: BTAM-104-18

M Code: 75362

Date of Examination : 06-06-2023

Time: 3 Hrs.

Max. Marks: 60

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY** consisting of **TEN** questions carrying **TWO** marks each.
2. **SECTION - B & C** have **FOUR** questions each, carrying **EIGHT** marks each.
3. Attempt any **FIVE** questions from **SECTION B & C**, selecting atleast **TWO** questions from each of these **SECTIONS B & C**.

SECTION-A

1. Write briefly :

a) State relation between Beta and Gamma functions.

b) Solve $\int_0^{\infty} \frac{x^8(1-x^6)}{(1+x)^{24}} dx$

c) Find the area of the region enclosed by the curve $y = x^2$ and the lines $x = 0$, $y = 0$ and $x = 2$.

d) Evaluate $\lim_{x \rightarrow 0} \frac{\log x^2}{\cot x^2}$

e) Calculate approximate value $\sqrt{26}$ to two decimal places by Taylor's theorem.

f) Are the vectors $(1,2,1)$, $(2,1,4)$, $(1,8,-3)$, $(4,5,6)$ linearly dependent? If yes, find relation between them.

g) Show that the transformation $T : R^2 \rightarrow R^3$ defined by $T(x,y,z) = (x+y, y+z, z+x)$ is linear.

h) Calculate A^5 for the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$.

i) Prove that the product of two orthogonal matrices of the same order is orthogonal.

j) State rank Nullity theorem.

SECTION-B

2. a) The curve $y^2(a+x) = x^2(3a-x)$ is revolved about the axis of x . Find the volume generated by the loop.

b) Find absolute maximum and minimum value of $f(x) = x - 2 \sin x$ on $[0, 2\pi]$.

3. a) Find rank of $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

b) Using properties of determinants, evaluate :

$$\begin{bmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{bmatrix}$$

4. a) Solve by Gauss Elimination method the system of equation

$$x+y+z=3, \quad 3x-9y+2z=-4, \quad 5x-3y+4z=6$$

b) Find the inverse of matrix

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

5. a) State and prove Rodrigue's Duplication formula.

b) Use Taylor's theorem to express $f(x) = 4x^3 - 5x^2 + 3x - 9$ in the powers of $(x-3)$.

SECTION-C

6. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x,y,z) = (3x+2y-4z, x-5y+3z)$

a) Find the matrix of T corresponding to bases of $\mathbb{R}^2$: $B_1 = \{(1,1,1), (1,1,0)\}$, $B_2 = \{(1,3), (2,5)\}$.

b) Verify that the action of T is preserved by its matrix representation i.e. $[T; B_1, B_2][v; B_1] = [T(v); B_2]$ for all $v \in \mathbb{R}^3$.

7. a) Express the matrix A as sum of symmetric and skew symmetric matrix where

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

- b) Examine whether A is similar to B where $A = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.

8. a) Find Eigen values & Eigen Vectors of $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

- b) Diagonalize $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

9. a) Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & -3 & -1 \\ -1 & 4 & 2 \\ -1 & 3 & 0 \end{bmatrix}$

- b) Let V be a vector space of 2×2 matrices over R and let $M = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$. Let $T: V \rightarrow V$ be the linear map defined by $T(A) = MA \forall A \in V$. Find the basis and dimension of

- i) Null space of T
- ii) Range of T.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.