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Total No. of Pages : 02

Total No. of Questions : 09

**B.Tech. (AI&ML/CSE/Data Science/AI/IOT/IT/CS/Internet of Things and Cyber Security including Block Chain Technology) (Sem.-4)**

**DISCRETE MATHEMATICS**

**Subject Code : BTCS-401-18**

**M.Code : 77626**

**Date of Examination : 08-12-2023**

**Time : 3 Hrs.**

**Max. Marks : 60**

**INSTRUCTIONS TO CANDIDATES :**

1. **SECTION-A is COMPULSORY** consisting of TEN questions carrying TWO marks each.
2. **SECTION-B** contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. **SECTION-C** contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

**SECTION-A**

**1. Write briefly:**

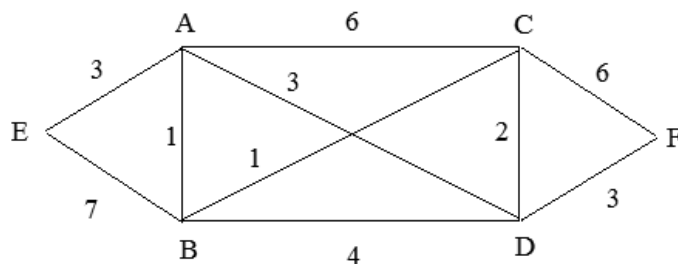
- a) Give an example of a relation which is reflexive, symmetric but not transitive
- b) Determine the domain and range of the relation  $R = \{(x,y) : x \in \mathbb{N}, x < 5, y = 3\}$
- c) How many 3- letter words can be made using the letters of the words "ORIENTAL"?
- d) State and Prove Idempotent Laws of Logic of Proposition.
- e) Define Monoid Groups.
- f) Find  $k$ , if a regular graph with 8 vertices has 12 edges.
- g) Define minimal spanning tree.
- h) Give an example of a connected graph that has both Euler circuit and Hamiltonian cycle.
- i) Define Chromatic Number.
- j) Define Equivalent Sets.

### SECTION-B

2. Let  $Z$  be the set of all integers and  $R$  be the relation on  $Z$  defined as  $R = \{(a,b): a,b \in Z \text{ and } (a-b) \text{ is divisible by } 5\}$ . Prove that  $R$  is an equivalence relation.
3.
  - a) How many people must you have to guarantee that at least 12 of them will have birthday on the same day of the week?
  - b) Find the number of positive integers from 1 to 1000 which are divisible by none of 5, 6 and 8.
4.
  - a) Prove that  $(p \leftrightarrow q) \leftrightarrow r = p \leftrightarrow (q \leftrightarrow r)$ .
  - b) Prove the validity of the following argument :
    - i) If it rains then crop will be good.
    - ii) It did not rain therefore the crop will not be good.
5. Prove that the order of a subgroup of a finite group divides the order of the group.
6. Show that a graph is a tree if and only if it is minimally connected.

### SECTION-C

7. Find shortest path from E to F using Dijkstra's algorithm for the following graph:



8. Show that the set  $G = \{0,1,2,3,4\}$  forms a field w.r.t addition and multiplication modulo 5.
9.
  - a) Give an example of a function (i) which is one to one but not onto, (ii) which is not one to one but onto.
  - b) Define the following function on integers by

$$f(k) = k + 1, g(k) = 2k \text{ and } h(k) = \left\lceil \frac{k}{2} \right\rceil$$

- i) Which of these are one to one?
- ii) Which of these are onto?

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**