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Total No. of Questions : 09

B.Tech. (AI & ML/DS)(CE)(CSE)(IT)(Robotics & Artificial Intelligence) (Internet of Things and Cyber Security including Block Chain Technology) (Sem.–1)

CALCULUS & LINEAR ALGEBRA

Subject Code : BTAM/104/18

M.Code : 75362

Date of Examination:02-01-2025

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B & C.**

SECTION-A

1. Solve the following :

a) Evaluate $\int_0^1 xe^{2x}\,dx$ using integration by parts.

b) Evaluate $\lim\limits_{x\to 0}\dfrac{\sin(2x)}{3x}$.

c) Verify the Mean Value Theorem for the function $f(x) = x^2 + 2x$ on the interval [1,4].

d) Calculate the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 2 \\ 7 & 9 & 6 \end{pmatrix}$.

e) Determine if the following column vectors are linearly independent : $u = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $v = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$, and $w = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$.

f) Identify the kernel of the linear transformation $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (x - y, 2x + y)$.

g) Show that the vectors (1, 0, 1), (2, 1, 3), (0, –1, 1) are Linearly independent in $\mathbb{R}^3$.

h) State rank-nullity theorem.

i) Find the eigenvalues of the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

j) What do you mean by similar matrices, give an example of two similar matrices.

SECTION-B

2. a) Find the critical points and determine the maximum and minimum values of $f(x) = \dfrac{4x}{1+x^2}$.

b) Expand $\dfrac{1}{1-x}$ using the Maclaurin series up to the third-degree term.

3. Solve the system of equations : $x + 2y - z = 4$, $3x - y + 2z = 5$, $2x + y + 3z = 7$ using Cramer's Rule.

4. If multiplication of two matrices $A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ a & b \end{pmatrix}$ is equal to $\begin{pmatrix} -1 & 6 \\ 2 & 8 \end{pmatrix}$, then find $a - 3b$.

5. State and prove the relationship between beta and gamma function.

SECTION-C

6. Find a basis for the subspace of $\mathbb{R}^3$ generated by the vectors (1, 2, 3) and (2, 4, 6).

7. Find the rank, nullity, kernel, arid matrix associated with the linear map $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (x - y, 3x + 4y)$.

8. Diagonalize the matrix $\begin{pmatrix} 4 & -3 & 0 \\ 2 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$.

9. Given the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, find the eigenvalues, eigenvectors, and determine if the eigenvectors form an orthogonal set.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.