

Roll No.

Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (AI&ML / AI&DS / Block Chain / CE / CSE / CS / DS / CSD / EE / ECE / EEE / ETE / IT / ME / Robotics & Artificial Intelligence / Internet of Things and Cyber Security including Block Chain Technology) (Sem.-2)

ENGINEERING MATHEMATICS-II

Subject Code : BTAM201-23

M.Code : 93811

Date of Examination : 10-01-2025

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

- 1. Answer the following :**

a) Using Echelon form, find the rank of the matrix $A = \begin{pmatrix} 3 & 1 & 7 \\ 1 & 2 & 1 \\ 4 & -1 & 7 \\ 4 & -1 & 5 \end{pmatrix}$.

b) Determine the eigen values of the matrix $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$.

- Let V be a vector space in $\mathbb{R}^3$. Examine whether $W = \{(a, b, c) : a^2 + b^2 + c^2 \leq 1\}$ is a subspace of V .
- Let V be a vector space over a field F . Show that any subset S of V containing zero vector is Linearly Dependent over F .
- Determine the value of k for which the system of equations :

$$x - ky + z = 0; \quad kx + 3y - kz = 0; \quad 3x + y - z = 0$$
has only trivial solution.

- (f) Verify if $y = \frac{x^2}{2} + C$ satisfies the differential equation $y' = x$.
- (g) Solve the initial value problem $\frac{dy}{dx} = x + 2$ given $y(0) = 1$.
- (h) Find an integrating factor for the differential equation $\frac{dy}{dx} + y \tan(x) = \sin(x)$.
- (i) Solve the nonlinear first-order PDE $\frac{\partial z}{\partial x} + z \frac{\partial z}{\partial y} = 0$.
- (j) Verify if the surfaces defined by $z = xy$ and $z = x^2 + y^2$ are orthogonal to each other at the point $(1, 1, 1)$.

SECTION-B

2. Solve the following system of equations using Gauss Elimination method :
- $$x - y + z = 4; \quad 2x + y - 3z = 0; \quad x + y + z = 2.$$
3. Find a Linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ whose range is spanned by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$.
4. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 3 & -1 \\ -2 & -1 & 1 \end{pmatrix}$. Find A^{-1} .
5. Find the inverse of the matrix using the Gauss-Jordan method : $\begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{pmatrix}$.

SECTION-C

6. Solve $y'' + y = \sin x$ using variation of parameters.

7. An electric RLC circuit has a resistance of $10 \, \Omega$, an inductance of $5 \, \text{H}$, and a capacitance of $0.02 \, \text{F}$. Form and solve the differential equation for the charge $Q(t)$ on the capacitor, given that $Q(0) = 0$ and $Q'(0) = 0$.
8. Solve the PDE $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} - z = 0$ using Lagrange's method, and verify your solution by checking if it satisfies the original equation.
9. Find the general solution of the linear PDE $u_{xxx} + 2u_{x^2y} + 2u_{xy^2} = e^x$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.