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Total No. of Pages : 02

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B.Tech.(AI & ML/AI & DS/CSE/DS/IOT/Internet of Things and Cyber Security including Block Chain Technology)) (Sem.-3)

MATHEMATICS-III

Subject Code : BTAM304-18

M.Code : 76438

Date of Examination : 25-06-2025

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY** consisting of **TEN** questions carrying **TWO** marks each.
2. **SECTION-B** contains **FIVE** questions carrying **FIVE** marks each and students have to attempt any **FOUR** questions.
3. **SECTION-C** contains **THREE** questions carrying **TEN** marks each and students have to attempt any **TWO** questions.

SECTION - A

1. **Solve the following :**

- Show that the function $f(x, y) = \begin{cases} \frac{xy}{x^2+2y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ is not continuous at $(0, 0)$.
- Change the order of integration of $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dy dx$.
- State Bolzano weistrass theorem.
- Find $\frac{du}{dt}$ when $u = x^2 + y^2$, $x = at^2$, $y = 2at$.
- State Cauchy's integral test for infinite series.
- Find the integrating factor of the differential equation $(y-1)dx - xdy = 0$.
- Write clairaut's equation.
- Solve $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$.
- Write Legendre's Homogenous linear equation.
- Use wronskian to show that the functions $x, x^2, x^3 (x \neq 0)$ are independent.

SECTION - B

2. If $\theta = t^n e^{-r^2/4t}$, Find the value of n which will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$.
3. Evaluate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} dx dy$ over the positive quadrant of the circle $x^2 + y^2 = 1$.
4. Discuss the convergence of the series $x \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots \infty$.
5. Solve by the method of variation of parameters $\frac{d^2 y}{dx^2} + 4y = \tan 2x$.
6. Solve $e^y y' = e^x (e^x - e^y)$.

SECTION - C

7. Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
8. a) **Examine the convergence of the series :**
$$1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \frac{1}{7^2} - \frac{1}{8^2} + \dots \infty.$$

b) Solve $p^2 + 2p \cot x = y^2$.
9. Solve $x^2 y'' - 4xy' + 8y = 4x^3 + 2 \sin(\log x)$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.