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Total No. of Questions : 09

B.Tech. (AI&ML/AI&DS/CE/CSE/IT/DS) (Internet of Things and Cyber Security including Block Chain Technology) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-104-18

M.Code : 75362

Date of Examination: 14-06-2025

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select at least TWO questions EACH from SECTION - B & C.**

SECTION - A

1. Explain briefly :

a) Verify Lagrange's Mean value theorem for $f(x) = e^x$ in [0, 1].

b) Show that sin x (1 + cos x) has a maximum when $x = \pi/3$.

c) Evaluate $\int_0^{\infty} x^2 e^{-x^2}\, dx$.

d) Determine the rank of the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

e) If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ verify that $AA^T = I = A^T A$, where I is the unit matrix.

f) For what values of k, do the following sets of vectors form a basis in $\mathbb{R}^3$: $\{(k, 1-k, k), (0, 3k-1, 2), (-k, 1, 0)\}$.

g) State $ran(T)$ and $ker(T)$ of a linear transformation $T : V \to W$. State rank-nullity theorem.

h) Examine whether the matrix A is similar to the matrix B where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

i) Express the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ -3 & 4 & -1 \\ -1 & 1 & 2 \end{bmatrix}$ as sum of symmetric and skew-symmetric matrices.

j) Show that the eigen values of symmetric matrix are real.

SECTION - B

2. a) Expand $\log(1 + x)$ by Maclaurin's series.

   b) Evaluate $\lim\limits_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$.

3. a) Find the surface area of the solid generated by the revolution of the astroid $x = a\cos^3 t,\ y = a\sin^3 t,$ about the y-axis.

   b) Prove that $\beta(m, n) = \int_0^1 \dfrac{x^{m-1} + x^{n-1}}{(1 + x)^{m+n}}\, dx.$

4. a) Solve the equations $3z + y + 2z = 3$, $2x - 3y - z = 3$, $x + 2y + z = 4$ by Cramer's rule.

   b) **Using the Gauss-Jordan method, find the inverse of the matrix :**

   $$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

5. a) Are the following vectors linearly dependent. If so, find the relation between them : $\{(1, 1, 1, 3), (1, 2, 3, 4), (2, 3, 4, 9)\}$.

   b) Investigate for what values of λ and μ, the simultaneous equations $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$, have no solution, a unique solution and an infinite number of solutions.

SECTION - C

6. a) Let T be a transformation from $\mathbb{R}^3$ into $\mathbb{R}^1$ defined by T $(x_1, x_2, x_3) = x_1^2 + x_2^3 + x_3^2$. Show that T is not a linear transformation.

   b) Find *ker* (T) and *ran* (T) and their dimensions for

$$T : \mathbb{R}^2 \to \mathbb{R}^3 ;\ T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x + y \\ y - x \\ 3x + 4y \end{pmatrix}.$$

7. a) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y + z \\ y - z \end{pmatrix}$.

   Determine the matrix of the linear transformation T, with respect to the standard basis

$$X = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \text{ in } \mathbb{R}^3 \text{ and } Y = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \text{ in } \mathbb{R}^2.$$

   b) If x, y, z and linearly independent vectors in $\mathbb{R}^3$ then show that $x + y, y + z, z + x$ are also linearly dependent in $\mathbb{R}^3$.

8. a) **Find the eigen values and the corresponding eigen vectors of the matrix :**

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}.$$

b) The eigen vectors of a 3 × 3 matrix A corresponding to the eigen values 2, 2, 4 are $(-2, 1, 0)^T$, $(-1, 0, 1)^T$, $(1, 0, 1)^T$ respectively. Find the matrix A.

9. Show that the matrix $A \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. Hence find P such that $P^{-1} AP$ is a diagonal matrix.

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